Caltech Division of the Humanities and Social Sciences

Ec 140 Border/Hoffman
Economic Progress Fall 2017

Midterm Examination

Trick or Treat: Due 2:30 pm, Tuesday, October 31

Instructions

This exam has one main question, with two parts. You have four hours total to answer it. It should not take that long if you think first.

The second question is to state whether you think we should keep papers on the reading list. This part of the exam is not timed.

All three parts carry equal weight.

You may not collaborate on this examination. You may use any notes that you have personally taken, downloaded from the official course web site, or copied by hand (in pencil or ink on paper). You may use any book in the Caltech library collection, if you think it will help. You may use Mathematica or a similar program, but you must explain what you asked it to do and why.

Write legibly in complete sentences and explain yourself.

1 Variations on a theme

Recall the simple growth model with two factors of production, capital K and labor L, constant returns to scale, and no technical change or depreciation, just population growth. It says

$$Y(t) = F\big(K(t), L(t)\big),$$

and

$$L(t) = L_0 e^{nt},$$

where n is the percentage rate of growth of the labor supply.

Let $k(t) = K(t)/L(t)$ denote the capital/labor ratio and $y(t) = Y(t)/L(t)$ denote the output per capita. Because of constant returns to scale we may write

$$y = f(k), \qquad \text{where } f(k) = F(k, 1).$$

We assume that a constant fraction s of output is invested in creating new capital, so

$$\dot{K}(t) = sY(t) = sf\big(k(t)\big)L(t). \tag{1}$$

We can also write $K(t) = k(t)L(t)$, which gives another expression for $\dot{K}$,

$$\dot{K}(t) = \dot{k}(t)L(t) + k(t)nL(t). \tag{2}$$

Equating (1) and (2) implies that the capital/labor ratio evolves according to the differential equation

$$\dot{k}(t) = sf\big(k(t)\big) - nk(t). \tag{3}$$

For a Cobb–Douglas production function,

$$Y = K^\alpha L^{1-\alpha}, \tag{4}$$

per capita income is given by

$$f(k) = k^\alpha. \tag{5}$$

The solution to the differential equation (3) for the Cobb–Douglas case is

$$k(t) = \left[\left(k_0^{1-\alpha} - \frac{s}{n}\right)e^{-n(1-\alpha)t} + \frac{s}{n}\right]^{\frac{1}{1-\alpha}}, \tag{6}$$

so that

$$k(t) \to \left(\frac{s}{n}\right)^{\frac{1}{1-\alpha}}. \tag{7}$$

You may wish to consult the on-line notes at http://people.hss.caltech.edu/~kcb/Courses/Ec140/Notes/SolowGrowth.pdf, which has an appendix on differential equations.

1.1 Variation I

Assume that there are three factors of production, physical capital K, human capital H, and raw labor L, and there is a Cobb-Douglas production function

$$Y = K^\alpha H^\eta L^{1-\alpha-\eta},$$

where $\alpha > 0$, $\eta > 0$, and $1 - \alpha - \eta > 0$. The rate of saving is fixed at s, but now it may be invested in either human or physical capital. Assume that at each instant t, the marginal product of K and H are the same. (You can justify this by assuming that investors will invest in whichever form of capital has the higher marginal product, and that the history is such that the marginal products are already equalized.)

1. What is the marginal product of K and of H?

2. What does this assumption of equality about marginal products imply about the relation between K and H?

3. Finish the analysis of the dynamics of this model. By this we mean that you should answer the following questions

   (a) Is there a nonzero steady state? That is, is there a value k^* such that $k(t) \equiv k^*$ satisfies the differential equation (3)?

   (b) If so, what is it? On what parameters does it depend? Is it unique? Is it stable?

   (c) If not, does growth in $y(t)$ continue positively, or is is it negative?

   (d) Ideally, you should write down a closed a form solution for $k(t)$, if you can.

1.2 Variation II

Assume that there are three factors of production, physical capital K, land A, and raw labor L, and there is a Cobb-Douglas production function

$$Y = K^{\alpha} A^{\gamma} L^{1-\alpha-\gamma},$$

where $\alpha > 0$, $\gamma > 0$, and $1 - \alpha - \gamma > 0$. The quantity of land is fixed and unalterable.

1. What is the crucial difference between this variation and the one above?

2. Analyze of the dynamics of this model.

2 Feedback

For each of these papers, indicate whether you think we should keep the paper on the reading list for the next time the course is offered, or whether we should drop it.

In addition, for your most favorite and least favorite papers, give a well-reasoned argument (a paragraph) for keeping or dropping them.

_____ Amartya Kumar Sen. Mortality as an indicator of economic success and failure. *Economic Journal*, 108(446):1–25, January 1998.

_____ Robert C. Allen. Progress and poverty in early modern Europe. *Economic History Review*, 56(3):403–443, August 2003.

_____ Robert M. Solow. Technical change and the aggregate production function. *Review of Economics and Statistics*, 39:312–320, August 1957.

_____ Jacob Schmookler. *Invention and Economic Growth*. Harvard University Press, 1966.

_____ William Easterly and Ross Levine. It's not factor accumulation: Stylized facts and growth models. *The World Bank Economic Review*, 15(2):177–219, 2001.

_____ Lant Pritchett. Divergence, big time. *Journal of Economic Perspectives*, 11(3):3–17, 1997.

_____ Robert E. Lucas, Jr. Why doesn't capital flow from rich to poor countries? *American Economic Review*, 80(2):92–96, May 1990.

_____ Robert C. Allen. *The British Industrial Revolution in Global Perspective*. New Approaches to Economic and Social History. Cambridge University Press, Cambridge, 2009.

_____ Morgan Kelly, Joel Mokyr, and Cormac Ó Gráda. Precocious Albion: A new interpretation of the British Industrial Revolution. 2013.

_____ Gregory Clark. Why isn't the whole world developed? Lessons from the cotton mills. *Journal of Economic History*, 47(1):141–173, March 1987.

_____ David S. Landes. Why Europe and the West? Why not China? *Journal of Economic Perspectives*, 20(2):3–22, Spring 2006.

Put your name here: __________________________