

## Second Half-term Examination

Due 3:00 pm, Friday, December 8

### Instructions

This exam has one main question. You have **three hours** total to answer it. It should not take that long if you think first.

The second question is to state whether you think we should keep papers on the reading list. This part of the exam is not timed.

Both parts carry equal weight.

You may not collaborate on this examination. You may use any notes that you have personally taken, downloaded from the official course web site, or copied by hand (in pencil or ink on paper). You may use any book in the Caltech library collection, if you think it will help. You may use Mathematica or a similar program, but you must explain what you asked it to do and why.

Write legibly in complete sentences and explain yourself.

## 1 Restating the main theme

Recall the simple growth model with two factors of production, capital  $K$  and labor  $L$ , constant returns to scale, and no technical change or depreciation, just population growth. It says

$$Y(t) = F(K(t), L(t)),$$

and

$$L(t) = L_0 e^{nt},$$

where  $n$  is the percentage rate of growth of the labor supply.

Let  $k(t) = K(t)/L(t)$  denote the capital/labor ratio and  $y(t) = Y(t)/L(t)$  denote the output per capita. Because of constant returns to scale we may write

$$y = f(k), \quad \text{where } f(k) = F(k, 1).$$

We assume that a constant fraction  $s$  of output is invested in creating new capital, so

$$\dot{K}(t) = sY(t) = sf(k(t))L(t). \quad (1)$$

We can also write  $K(t) = k(t)L(t)$ , which gives another expression for  $\dot{K}$ ,

$$\dot{K}(t) = \dot{k}(t)L(t) + k(t)nL(t). \quad (2)$$

Equating (1) and (2) implies that the capital/labor ratio evolves according to the differential equation

$$\dot{k}(t) = sf(k(t)) - nk(t). \quad (3)$$

For a Cobb–Douglas production function,

$$Y = K^\alpha L^{1-\alpha}, \quad (4)$$

per capita income is given by

$$f(k) = k^\alpha. \quad (5)$$

**Fact:** The solution to the differential equation

$$\dot{x}(t) = ax(t)^b - cx(t) \quad (6)$$

where  $a, b, c > 0$ ,  $b < 1$ , with initial condition  $x(0) = x_o$  is

$$x(t) = \left[ \left( x_o^{1-b} - \frac{a}{c} \right) e^{-c(1-b)t} + \frac{a}{c} \right]^{\frac{1}{1-b}} \quad (7)$$

Consequently, there is a nonzero steady state

$$x^* = \left( \frac{a}{c} \right)^{\frac{1}{1-b}} \quad (8)$$

and

$$x(t) \xrightarrow[t \rightarrow \infty]{} \left( \frac{a}{c} \right)^{\frac{1}{1-b}} \quad (9)$$

provided  $x_o > 0$ .

### Variation III

Now imagine that **ideas** are the source of economic growth because they open the door to increasing returns, but ideas have to be shared for that to be possible. Imagine too that a technology appears that suddenly makes it possible for a larger fraction  $\gamma$  of the population to share ideas. It might be CAD/CAM, virtual reality, or even urbanization, if it facilitates learning by doing. The following model might capture what would happen to GDP  $Y$  if  $\gamma$  were to increase (here  $1 \geq \gamma \geq 0$ ):

$$Y = L^\gamma K^\alpha L^{1-\alpha} \quad (10)$$

Here  $L$  is the labor supply (equal to the total population), which grows at a constant rate  $n$ ;  $K$  is the capital stock; and  $0 < \alpha < 1$ . The first term  $L^\gamma$  captures the effect of sharing ideas among a fraction  $\gamma$  of the population. Assume that everyone saves a constant fraction  $s$  of output, which is added to the capital stock.

1. Show that the aggregate production defined by equation (10) does not have constant returns to scale.
2. Transform equation (10) into one that is constant returns to scale by defining the idea-enhanced labor supply  $I$  and capital per idea-enhanced worker  $K/I$ .
3. Write down the equation for the rate of change of  $K/I$ .
4. What does  $K/I$  converge to? What does the **growth rate** of  $Y/I$  converge to?
5. Our interest, though, is not the growth rate of  $Y/I$  but the growth rate of  $Y/L$ . What does the growth rate of  $Y/L$  converge to?
6. Suppose that virtual reality technology changes  $\gamma$  from 0 to 0.5. How does the long run growth rate of  $Y/L$  change?
7. Is there anything in the course readings that might support such a model? If so, what might some obstacles to raising  $\gamma$  be?

## 2 Feedback

For each of these papers, indicate whether you think we should keep the paper on the reading list for the next time the course is offered, or whether we should drop it.

In addition, for your most favorite and least favorite papers, give a well-reasoned argument (a paragraph) for keeping or dropping them.

\_\_\_\_\_ Charles I. Jones and Paul M. Romer. The new Kaldor facts: Ideas, institutions, population, and human capital. Working Paper 15094, National Bureau of Economic Research, Cambridge, MA, 2009.

\_\_\_\_\_ Paul M. Romer. Why, indeed, in America? theory, history, and the origins of modern economic growth. *American Economic Review*, 86(2):202–206, May 1996.

\_\_\_\_\_ Robert E. Lucas, Jr. Making a miracle. *Econometrica*, 61(2):251–272, March 1993.

\_\_\_\_\_ Oded Galor and David N. Weil. Population, technology, and growth: From Malthusian stagnation to the demographic transition and beyond. *American Economic Review*, 90(4):806–828, September 2000.

\_\_\_\_\_ Enrico Spolaore and Romain Wacziarg. How deep are the roots of economic development? *Journal of Economic Literature*, 51(2):325–369, 2013.

\_\_\_\_\_ George J. Stigler and Gary S. Becker. De gustibus non est disputandum. *American Economic Review*, 67(2):76–90, March 1977.

\_\_\_\_\_ Daron Acemoglu, Simon Johnson, and James A. Robinson. Reversal of fortune: Geography and institutions in the making of the modern world income distribution. *Quarterly Journal of Economics*, 117(4):1231–1294, November 2002.

\_\_\_\_\_ Edward L. Glaeser, Rafael La Porta, Florencio Lopez-De-Silanes, and Andrei Shleifer. Do institutions cause growth? *Journal of Economic Growth*, 9(3):271–303, September 2004.

\_\_\_\_\_ Nathan Nunn. The long-term effects of Africa’s slave trades. *Quarterly Journal of Economics*, 123(1):139–176, February 2008.

\_\_\_\_\_ Melissa Dell. The persistent effects of Peru’s mining *Mita*. *Econometrica*, 78(6):1863–1903, November 2010.

\_\_\_\_\_ Andrei Shleifer and Robert W. Vishny. Corruption. *Quarterly Journal of Economics*, 108(3):599–617, August 1993.

\_\_\_\_\_ Abhijit V. Banerjee and Esther Dufo. Growth theory through the lens of development economics. In Philippe Aghion and Steven N Durlauf, editors, *Handbook of Economic Growth*, pages 473–552. Elsevier, Amsterdam, 2005.

\_\_\_\_\_ Nicholas Bloom, Charles I. Jones, John Van Reenen, and Michael Webb. Are ideas getting harder to find? Preprint., September 2017.

\_\_\_\_\_ Robert James Gordon. *The Rise and Fall of American Growth: The U.S. Standard of Living Since the Civil War*. The Princeton Economic History of the Western World. Princeton University Press, 2016.

Put your name here: \_\_\_\_\_